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A simple universal adaptive feedback controller for chaos and hyperchaos control

Jinling Zheng

Basic Department, Shandong Urban Construction Vocational College, Jinan 250014, PR China

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ABSTRACT

A simple universal adaptive feedback controller is proposed for chaos control. In comparison with previous methods, the proposed scheme, which uses a single feedback gain and converges very fast, is suitable for application to a larger class of chaotic, hyperchaotic and nonhyperbolic chaotic systems. A sufficient condition for selecting the least feedback terms is given, and a numerical example using the Lorenz system verifies the correctness and effectiveness of the proposed approach.

1. Introduction

A chaotic system has complex dynamical behaviors, such as depending sensitively on tiny variations of initial conditions, having bounded trajectories in the phase space, etc. This complex behavior could be undesirable in many biological, physical and engineering applications, due to the fact that long-term prediction is impossible for such systems. For this reason, it is often necessary to design control mechanisms that will force a system to exhibit a desired dynamics, even when intrinsically chaotic. This has been the focus of a large body of work in recent times [1–3]. In particular, designing efficient mechanisms for realizing the goal of chaos control using simple and physically available controllers is very significant for both theoretical research and practical applications [4]. In 1990, the celebrated OGY control scheme was presented by Ott, Grebogi and Yorke [1]. This method has been found very effective for controlling a large number of chaotic systems. The OGY method seeks to use small perturbations to place chaotic orbits onto desired (unstable) periodic orbits. Due to the ergodicity of the chaotic orbit on the attractor, they are eventually driven to the desired periodic orbit and thus can be captured by a small control.

However, a careful study of the OGY control scheme has revealed some limitations both theoretically and experimentally [5–7]; and particularly for nonhyperbolic systems for which prior knowledge of the dynamics is unavailable. For instance, in Ref. [5], the failure of the OGY scheme for a weakly perturbed pendulum was demonstrated; while in Ref. [6], some problems associated with the application of the OGY-type controls for chaos were theoretically analyzed. Evidently, in the OGY method, chaos is destroyed before obtaining the required information necessary for successful control. Since many physically chaotic systems are actually nonhyperbolic, the question of how to design a universal and effective control algorithm and, hence, to overcome the limitation of the OGY control is a crucial one.

Although the adaptive control method [8–11] is an effective way to control chaotic systems, in most research into chaos control, controller designs appear purely arbitrary and complex, and stabilization of the chaotic systems can only be realized from a mathematical viewpoint. There is no link between the controllers and the physical configuration of the electronic set-up. From the control theory viewpoint, the real problem for stabilization of the chaotic systems is designing a feedback scheme such that less possibly simple controllers are interconnected. Each controller is based on the adaptive feedback control strategy; thus, stabilization of the chaotic systems can be achieved by electronic realization.

E-mail address: jinling_zheng@163.com.

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Very recently, Huang [7] made an attempt to address this problem on the basis of an adaptive control approach. The method proposed by Huang is simple to implement in practice and quite robust against the effect of noise. This technique has also been extended to the synchronization of identical chaotic systems [12] and has been adopted by some authors in order to realize the identical synchronization of almost all kinds of coupled identical neural networks with time-varying delay, complete synchronization in uncertain complex networks [13] and adaptive projective synchronization of unified chaotic systems [14]. However, this method also has two disadvantages. The first is that the number of the feedback gains ($\epsilon_i, i = 1, 2, \dots, n$) required to achieve chaos control in Huang's control scheme is equal to the dimension of the chaotic system, which adds to the difficulty of stabilization of chaotic orbits in the chaotic system. In particular, if the dimension of the chaotic system is high, such as for hyperchaotic systems, achieving stabilization of chaotic orbits in the chaotic system becomes more difficult than for the low dimension chaotic systems. A natural question is that of whether this controller can use only one feedback gain. The other disadvantage of Huang's method is how to select the feedback terms which are needed in the controller u to stabilize the chaotic orbits in the chaotic system. To possibly address this shortcoming, it was assumed in Ref. [7,15] that the controller does not require all the feedback terms ($x_i, i = 1, 2, \dots, n$) in some experiments and that the feedback control when added to only partial variables is sufficient to stabilize chaotic orbits in the chaotic system. But the approach of selecting only one feedback term as described in Ref. [7,15] does not always lead to stabilization. For instance, setting $x_i = 0$ (if $|x_i| < |x_j|$), and thus canceling the corresponding coupling, is not feasible for the famous Lorenz system and its member family as we shall show in this paper.

In this paper, we give a novel answer to the above open problem. By using the LaSalle invariance principle, we obtain a universal adaptive feedback controller in the sense that the control, when applied to any member of the chaotic systems, ensures that: (i) the zero state of the controlled dynamical system is globally attractive; and (ii) the adaptive feedback gain converges to a finite limit. Then, we prove that an adaptive feedback controller with only one feedback gain and the least feedback terms can strictly stabilize orbits of chaotic systems to the equilibrium point. The present control scheme is not only simple but also suitable to apply to all chaotic systems. In particular, for three-dimensional chaotic systems the controller can include just one feedback term, x_i.

2. Main results

In this section, we establish a novel, simple and universal adaptive feedback controller for a class of chaotic systems, and then propose the main results of this paper.

Consider a chaotic system be given as

$$\dot{x} = f(x), \tag{1}$$

where $x = (x_1, x_2, \dots, x_n)^T \in \mathrm{R}^n$, $f(x) = (f_1(x), f_2(x), \dots, f_n(x))^T : \mathrm{R}^n \to \mathrm{R}^n$ is a smooth vector function, i.e., $f(x) \in C^1$. Without loss of generality, let $\Omega \subset \mathrm{R}^n$ be a chaotic bounded set of (1) which is globally attractive, and suppose that $x_e = 0$ is an unstable equilibrium point embedded in Ω. Stabilizing the chaotic orbits in the system (1) to its equilibrium $x_e = 0$ is assumed to be the target control goal. For the vector function $f(x)$, we give the following general assumption:

Assumption 1. For any $x = (x_1, x_2, \dots, x_n)^T \in \Omega$, there exists a constant $l > 0$ satisfying

$$|f_i(x)| \le l|x|_\infty^{2n-1}, \quad i = 1, 2, \dots, n, \tag{2}$$

where $|x|_\infty$ is the ∞-norm of x, i.e., $|x|_\infty = \max_j \mid x_j \mid, j = 1, 2, \dots, n$.

Remark 1. This condition is extraordinarily weak compared with the uniform Lipschitz condition in [7]. Therefore the class of systems in the form of (1) and (2) includes almost all well-known finite-dimensional chaotic and hyperchaotic systems.

In order to stabilize the chaotic orbits in (1) to the equilibrium point $x_e = 0$, we add the following adaptive feedback controller u to the chaotic system (1):

$$\dot{x}_i = f_i(x) + u_i = f_i(x) + k_1\delta_{ij}x_i, \quad i = 1, 2, \dots, n, \tag{3}$$

where $u = (u_1, u_2, \dots, u_n)^T = k_1(\delta_{1j}x_1, \delta_{2j}x_2, \dots, \delta_{nj}x_n)^T$ is the controller, and $\delta_{ij} = 1$ if and only if $i = j$, $\delta_{ij} \equiv 0$ as long as $i \neq j$. The values of the δ_{ij} ($i = 1, 2, \dots, n$) will be determined by the following Condition 1. Significantly different from the usual linear feedback controller case, the single feedback gain k_1 is adapted according to the following update law:

$$\dot{k}_1 = -\gamma \sum_{j=i}^{n} \delta_{ij}(x_i - 0)^{2n} = -\gamma \sum_{j=i}^{n} x_i^{2n}, \tag{4}$$

where γ is an arbitrary positive constant.

For the system (3) and (4), we introduce a positive definite function:

$$V = \frac{1}{2}\sum_{i=1}^{n} x_i^2 + \frac{1}{2}\frac{1}{\gamma}(k_1 + L)^2, \tag{5}$$

where L is large positive constant, i.e., $L \geq nl|x|_\infty^{2n}/(\sum_{i=j}^{n}|x_i|^{2n})$. The feedback states x_i ($j = i, i = 1, 2, \ldots, n$) which are needed in the controller u are chosen on the basis of the following condition:

Condition 1. According to $\dot{V}(x) = 0$, i.e., $\sum_{i=1}^{n} x_i f_i(x) = L\sum_{i=1}^{n} \delta_{ij} x_i^{2n}$, if the states $x_i = 0, i \in \{1, 2, \ldots, n\}$, which also makes the states $x_k = 0, k \neq i, k \in \{1, 2, \ldots, n\}$, then we let $\delta_{ij} = 1$ and $\delta_{kj} = 0$.

Remark 2. According to the Condition 1, it is easy to obtain that $\dot{V}(x) = 0$, i.e., $\sum_{i=1}^{n} x_i f_i(x) = L\sum_{i=j}^{n} x_i^{2n}$, and $E = \{(x_1, x_2, \ldots, x_n, k_1)|\dot{V}(x) = 0\} = \{0\}$ because of the fact that the states $x_i = 0$ ($i \in \{1, 2, \ldots, n\}$) which also makes the states $x_k = 0$ ($k \neq i, k \in \{1, 2, \ldots, n\}$).

Then, we give the following main result.

Theorem 1. *Starting from any initial values of system (3) and (4), the orbits $(x(t), k_1(t))^T$ converge to $(0, k_0)^T$ as $t \to \infty$, where k_0 is a negative constant depending on the initial value. This implies that the adaptive feedback controller stabilizes the chaotic orbits to the equilibrium point $x_e = 0$.*

Proof. By differentiating the function V along the trajectories of the system (3) and (4), according to Assumption 1 and Remark 2, we obtain

$$\begin{aligned}
\dot{V} &= \sum_{i=1}^{n} x_i \dot{x}_i + \frac{1}{\gamma}(k_1 + L)\dot{k}_1 \\
&= \sum_{i=1}^{n} x_i (f_i(x) + k_1 \delta_{ij} x_i) - (k_1 + L)\sum_{j=i}^{n} x_i^{2n} \\
&= \sum_{i=1}^{n} x_i f_i(x) - L\sum_{j=i}^{n} x_i^{2n} \\
&\leq nl|x|_\infty^{2n} - L\sum_{j=i}^{n} x_i^{2n} \leq 0. \qquad (6)
\end{aligned}$$

It is clear that the set $E = \{(x_1, x_2, \ldots, x_n, k_1)|\dot{V}(x) = 0\} = \{0\}$ is the largest invariant set for the system (3) and (4). According to the well-known LaSalle invariance principle, $x_i(t) \to 0$ and $k_1 \to k_0$ as $t \to \infty$. Therefore, the proof of Theorem 1 is completed. □

This method can be easily used in practice by simply following the steps below:

I. Choose the feedback terms x_i ($i \in \{1, 2, \ldots, n\}$) which are needed in the controller u according to Condition 1, that is to say, we obtain that $\delta_{ij} = 1, \delta_{kj} = 0, k, i \in \{1, 2, \ldots, n\}$.
II. Add the controller $u = (u_1, u_2, \ldots, u_n)^T$ to the chaotic system (1), where $u_i = k_1 x_i, u_k = 0, k \neq i$ and $k, i \in \{1, 2, \ldots, n\}$. Then, set $\dot{k}_1 = -\gamma\sum_{j=i}^{n} x_i^{2n}$.

By these two steps, stabilization of the chaotic system is realized.

Remark 3. If $x_e \neq 0$ is an equilibrium point of the chaotic system, this method is also easily applicable by carrying out a coordinate transformation.

3. Illustrative examples

In this section, we take a Lorenz system as an example to show how to use the results obtained in this paper to stabilize chaotic orbits in a chaotic system.

Example 1. We consider the Lorenz system

$$\begin{aligned}
\dot{x}_1 &= a(x_2 - x_1), \\
\dot{x}_2 &= bx_1 - x_2 - x_1 x_3, \qquad (7) \\
\dot{x}_3 &= -cx_3 + x_1 x_2
\end{aligned}$$

where $a = 10,\ b = 28,\ c = \frac{8}{3}$.

Remark 4. We select the initial states values of the Lorenz system (7) as $x_1(0) = 0.1, x_2(0) = 0.2, x_3(0) = 0.3$; Fig. 1 shows the time evolution of the Lorenz system without control. We can set $x_3 = \max_i \mid x_i \mid, i = 1, 2, 3$, and add the controller

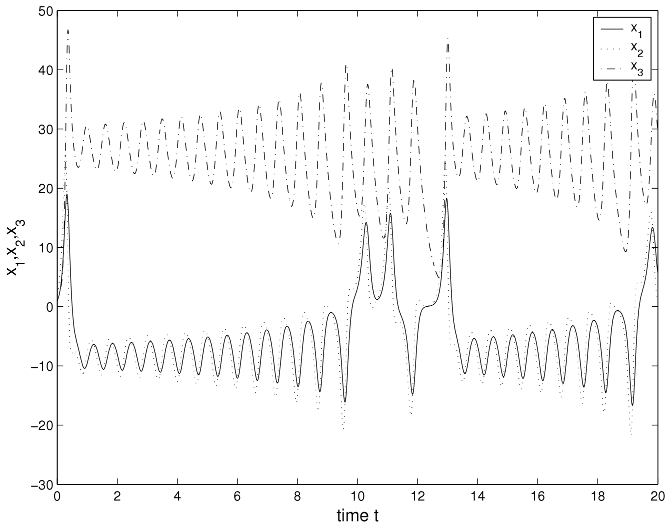

Fig. 1. Selecting the initial states values of the Lorenz system as $x_1(0) = 0.1, x_2(0) = .0.2, x_3(0) = 0.3$, here we show the time evolution of the Lorenz system without control; we can see that $x_3 = \max_i \mid x_i \mid, i = 1, 2, 3$.

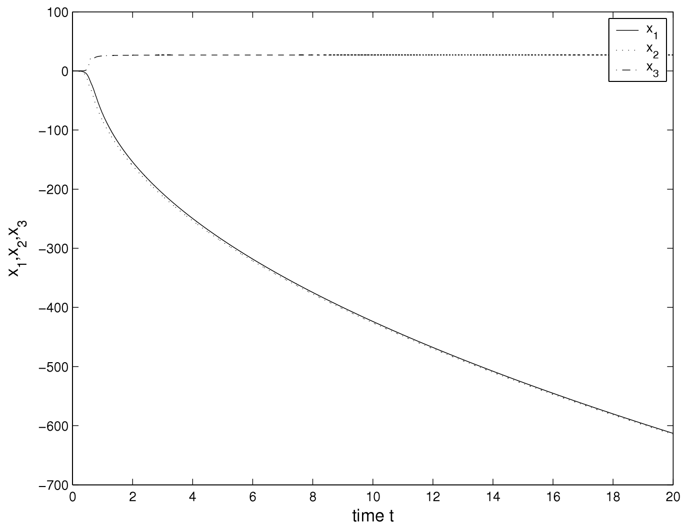

Fig. 2. Selecting the initial states values of the Lorenz system (7) as $x_1(0) = 0.1, x_2(0) = .0.2, x_3(0) = 0.3$ and the initial value of the controller as $k_1(0) = -1$, and selecting $\gamma = 10$, chaos control is not realized by the controller $k_1x = (0, 0, k_1x_3)^T$.

$k_1x = (0, 0, k_1x_3)^T$ to the Lorenz system. But Fig. 2 shows that chaos control is not realized, implying that the controller proposed in [7] when we set $x_i = 0$ (note that this cancels the corresponding coupling), if $|x_i| < |x_j|$ [7], is not feasible, as can be seen in Fig. 2.

Then we use our method to stabilize the chaotic orbits in (7) to the equilibrium point $x_e = 0$. According to Condition 1, $\dot{V}(x) = 0$, i.e., $38x_1x_2 = 10x_1^2 + x_2^2 + \frac{8}{3}x_3^2 + L\sum_{j=i}^{3}\delta_{ij}x_i^2$. Obviously, if $x_1 = 0$ (on the left hand side of the above equation), then $x_2 = 0, x_3 = 0$ (the right hand side of the above equation); thus, $E = \{(x_1, x_2, x_3)|\dot{V}(x) = 0\} = \{0\}$. So, we can select the controller $k_1x = (k_1x_1, 0, 0)^T$ and $\dot{k}_1 = -10x_1^2$ (selecting $\gamma = 10$). Next, we give the numerical verification of the above theoretical results, selecting the initial state values of the system (7) as stated above. With the initial value of the controller $k_1(0) = -1$, Fig. 3 shows that the Lorenz system is asymptotically stable to the zero solution when $t \to \infty$ and the feedback gain k_1 tends to a negative constant.

4. Conclusion

To summarize, a simple adaptive feedback control that is capable of realizing chaos control for almost all well-known chaotic systems has been obtained in this paper. Unlike the previous methods, the present controller method is not only

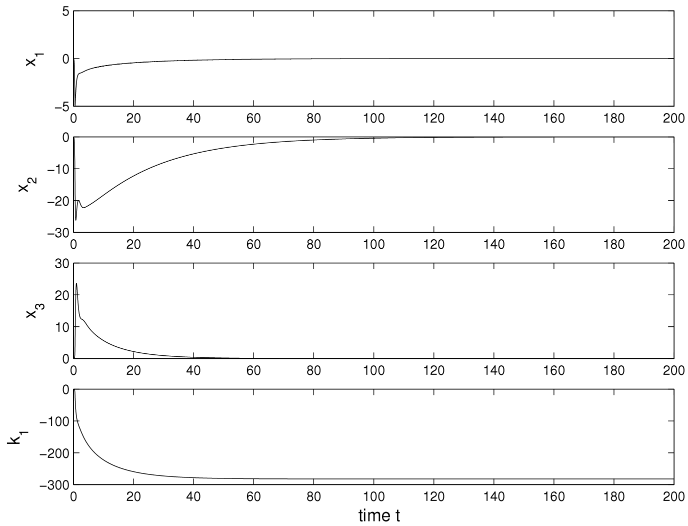

Fig. 3. The Lorenz system is asymptotically stable to zero when $t \to \infty$ and the feedback gain k_1 tends to a negative constant.

simple but also can be generalized. The condition for selecting the least feedback terms has been given. This controller can in particular include just one feedback term x_i for three-dimensional chaotic systems. Therefore, it could be easily implemented in practice. Moreover, the control idea could be generalized to the case of discrete chaotic systems. We believe that such a simple adaptive controller will be very beneficial for experimental applications in chaos control, especially where the OGY and Huang approaches fail.

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